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Time dependent supergravity solutions in arbitrary dimensions

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Abstract: By directly solving the equations of motion we obtain the time dependent solutions of supergravities with dilaton and a q-form field strength in arbitrary dimensions. The metrics are assumed to have the symmetries $\mathrm{ISO}(p+1) \times \mathrm{SO}(d-p-2,1)$ and can be regarded as those of the magnetically charged euclidean or space-like branes. When we impose the extremality condition, we find that the magnetic charges of the branes become imaginary and the corresponding real solutions then represent the Ep-branes of type II* theories (for the field-strengths belonging to the RR sector). On the other hand, when the extremality condition is relaxed we find real solutions in type-II theories which resemble the solutions found by Kruczenski-Myers-Peet. In $d = 10$ they match exactly. We point out the relations between the solutions found in this paper and those of Chen-Gal'tsov-Gutperle in arbitrary dimensions. Although there is no extremal limit for these solutions, we find another class of solutions, which resemble the solutions in the extremal case with imaginary magnetic charges and the corresponding real solutions can be regarded as the non-BPS Ep-brane solutions of type-II* theories (for the field-strengths in RR sector).

Keywords: Superstrings and Heterotic Strings, p-branes, D-branes, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

Low energy effective actions of (dimensionally reduced) string/M theories are known [1]–[4] to admit various kinds of time dependent solutions. These solutions are interesting from the cosmological point of view [3, 4] and are believed to shed light on the dS/CFT correspondence [5]–[8]. One particular class of solutions which might be useful in this context were assumed in the literature to have the symmetry $\mathrm{ISO}(p+1) \times \mathrm{SO}(d-p-2,1)$ of the metric in d space-time dimensions and can be regarded as those of magnetically charged euclidean or Space-like branes [9] (Sp-branes[1] for short). These solutions were found by Kruczenski-Myers-Peet (KMP) in $d = 10$ [10] and by Chen-Gal'tsov-Gutperle (CGG) in arbitrary d [11] in two different coordinate systems. Though in $d = 10$, these two solutions can be shown [12] to be equivalent by a coordinate transformation, the two coordinate systems have the advantages of their own. So, for example, the solutions in CGG coordinates were found to be useful to obtain a four-dimensional FRW universe with accelerating cosmology [13, 14], whereas, the KMP coordinates are more suitable to understand the dS/CFT correspondence.

In [10], KMP obtained the supergravity S-brane solutions in $d = 10$, starting from the known solutions [15] of vacuum Einstein equation in $d = 11$ with appropriate symmetries and using a rotation, compactification to $d = 10$ as well as applying T-duality symmetries [16, 17]. This way one could avoid solving the non-linear differential equations resulting from Einstein's equations. In this paper, we directly solve the non-linear equations of motion (in KMP coordinates) of the supergravities containing the dilaton and an abitrary rank field-strength to obtain the S-brane solutions in d dimensions. We assume the metrics to have the symmetries $\mathrm{ISO}(p+1) \times \mathrm{SO}(d-p-2,1)$ and therefore obtain only the localized and isotropic (as opposed to ref. [10], where anisotropic S-brane solutions were

[1] Sp-branes by definition have $(p+1)$-dimensional euclidean world-volume.

also found) S-brane solutions. One advantage of this direct method is that one can see what are the other kinds of solutions the equations of motion admit. In fact, we find that if the extremality condition (similar to the static BPS p-brane solutions [18]) is assumed to hold, the magnetic charges of the branes become imaginary. This implies that the corresponding real solutions represent Ep-branes of type II* theories when the field-strengths belong to RR sector. The 'starred' theories were introduced by Hull [19] in studying the compactifications of type-II string theories on a time-like circle.[2] Actually, it was shown there that type-IIA (IIB) string theory compactified on a time-like circle of radius R is equivalent to type-IIB* (IIA*) string theory compactified on a dual time-like circle of radius $1/R$. The 'starred' theories have a number of problems as emphasized by Hull in [19]. However, if time-like T-duality is consistent in string theories, then 'starred' theories are also consistent as string theories. It was pointed out in [19] that 'starred' string theories truncated to the supergravity level may give rise to theories with ghosts, but the full string theories would be ghost-free as they are equivalent to the ordinary type-II theories by a T-duality (time-like) transformation. On the other hand, if the extremality condition is relaxed, then real solutions can be found in type-II theories and the equations of motion can be solved consistently if we introduce at least three unknown parameters.[3] These solutions resemble the solutions obtained by KMP in [10]. In $d = 10$ they match exactly with the KMP solutions for the isotropic case. We also point out how these solutions in arbitrary d are related to the solutions found by CGG in ref. [11] by a coordinate transformation. We note that there is no extremal limit for these solutions. However, there exist another class of solutions for which the magnetic charges are imaginary and resemble the solutions in the extremal case and we point out that the corresponding real solutions represent the non-BPS Ep-branes of type-II* theories again for the field-strengths in RR sector.

This paper is organized as follows. In section 2, we obtain the time dependent solution of the supergravity equations of motion with the metrics having the symmetries ISO$(p+1) \times$ SO$(d-p-2, 1)$ and the components satisfying the extremality condition. Here we obtain the euclidean brane solutions with imaginary magnetic charges, representing the Ep-branes of type-II* theories. In section 3, we relax the extremality condition and obtain real time dependent solutions in type-II theories characterized by three parameters. In subsection 3.1, we show that for $d = 10$, our solutions match exactly with those obtained by KMP in [10]. In subsection 3.2, we show how for arbitrary d our solutions are related to those of CGG by a coordinate transformation similar to that found in [12]. In section 4, we discuss another class of solution which is non-extremal but is very similar to the extremal solutions found in section 2 and the magnetic charges are imaginary. We point out that these solutions can be interpreted as the non-BPS Ep-brane solutions of type-II* theories. We conclude our paper in section 5.

[2]The actions for the 'starred' theories can be obtained from those of the orinary theories by changing the signs of the kinetic terms of the RR sector gauge fields, but the kinetic terms of the NSNS sector gauge fields do not change sign.

[3]Here we also restrict our metrics to become asymptotically flat (in Rindler coordinates) and the dilaton to approach unity. If we do not impose this restriction, then our solutions will be characterized by an additional parameter.

2. General extremal euclidean brane solutions

In this section we discuss the euclidean brane[4] solutions of supergravity equations of motion in d-dimensions when the metric components satisfy an extremality condition similar to the static BPS p-brane solutions. The d-dimensional action of a graviton, dilaton and a q-form field-strength with dilaton coupling a in Einstein frame has the form,

$$S = \int d^d x \sqrt{-g} \left[R - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2 \cdot q!} e^{a\phi} F_{[q]}^2 \right] . \tag{2.1}$$

The above action is quite general and consists of the bosonic sector of (dimensionally reduced) string/M theories. The field-strength in (2.3) is real, but as pointed out in [19], it could be imaginary if, for example, it belongs to RR sector of type-II theory. For the latter case, the action could be obtained from the so-called 'starred' theory with the real field-strength but the kinetic term will have opposite sign from that of ordinary theory.

The equations of motion following from (2.1) have the forms,

$$R_{\mu\nu} - \frac{1}{2}\partial_\mu \phi \partial_\nu \phi - \frac{e^{a\phi}}{2(q-1)!} \left[F_{\mu\alpha_2 \ldots \alpha_q} F_\nu{}^{\alpha_2 \ldots \alpha_q} - \frac{q-1}{q(d-2)} F_{[q]}^2 g_{\mu\nu} \right] = 0 \tag{2.2}$$

$$\partial_\mu \left(\sqrt{-g} e^{a\phi} F^{\mu\alpha_2 \ldots \alpha_q} \right) = 0 \tag{2.3}$$

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} \partial^\mu \phi \right) - \frac{a}{2 \cdot q!} e^{a\phi} F_{[q]}^2 = 0 \, . \tag{2.4}$$

We will solve the above equations with the following ansatz,

$$ds^2 = e^{2A(t)} \left(-dt^2 + t^2 dH_{d-p-2}^2 \right) + e^{2B(t)} \left(dx_1^2 + \cdots + dx_{p+1}^2 \right) \tag{2.5}$$

$$F_{[q]} = b \, \mathrm{Vol}(H_{d-p-2}) \, . \tag{2.6}$$

Here 't' is a time-like coordinate and A, B are functions of 't' whose forms will be obtained by solving the equations of motion (2.2)–(2.4). dH_{d-p-2}^2 is the line element of a unit $(d-p-2)$ dimensional hyperbolic space and $\mathrm{Vol}(H_{d-p-2})$ is its volume-form. $q = d-p-2$ is the rank of the field-strength and b is the magnetic charge. Note that the metric in (2.5) has the symmetry $\mathrm{ISO}(p+1) \times \mathrm{SO}(d-p-2,1)$. The Ricci tensor of the hyperbolic space is given as $\bar{R}_{ab} = -(q-1)\bar{g}_{ab}$, with $\bar{g}_{ab}$ being its metric. Also the metric has the form of an euclidean brane with $(p+1)$-dimensional world-volume whose transverse space metric, upto a conformal factor $e^{2A(t)}$, can be written as,

$$\begin{aligned} -dt^2 + t^2 dH_{d-p-2}^2 &= -dt^2 + t^2 d\psi^2 + t^2 \sinh^2 \psi d\Omega_{d-p-3}^2 \\ &= -d\tilde{t}^2 + dr^2 + r^2 d\Omega_{d-p-3}^2 \end{aligned} \tag{2.7}$$

with $d\Omega_{d-p-3}^2$ being the line element of unit $(d-p-3)$-dimensional sphere. In the last expression of (2.7) we have used $t^2 = \tilde{t}^2 - r^2$ and $\tanh \psi = r/\tilde{t}$. This shows that the transverse space is flat upto a conformal factor in $\tilde{t}$, r coordinates and the metric has the required symmetry, since $e^{2A(t)}$ is invariant under $\mathrm{SO}(d-p-2,1)$. We will also

[4] Euclidean branes in type-II theories are termed as S-branes, whereas euclidean branes in type II* theories are termed as E-branes for the field-strengths belonging to RR sector.

assume that $A(t)$, $B(t)$ will vanish as $t \to \infty$, i.e. the metric is asymptotically flat (in Rindler coordinates). So, (2.5) and (2.6) represent the magnetically charged euclidean branes. The corresponding electrically charged branes can be obtained by using $g_{\mu\nu} \to g_{\mu\nu}$, $F \to e^{-a\phi} * F$, $\phi \to -\phi$, where $*$ is the d-dimensional Hodge dual.

Since eq. (2.3) is satisfied with the ansatz (2.6), we will solve (2.2) using the ansatz (2.5), (2.6). The non-zero components of the Ricci tensor are given below,

$$\begin{aligned} R_{tt} &= -(p+1)\left[\ddot{B} + \dot{B}^2 - \dot{A}\dot{B}\right] - q\left[\ddot{A} + \frac{\dot{A}}{t}\right] \\ R_{xx} &= e^{2B-2A}\left[\ddot{B} + (q-1)\dot{A}\dot{B} + (p+1)\dot{B}^2 + q\frac{\dot{B}}{t}\right] \\ R_{ab} &= t^2\left[\ddot{A} + (q-1)\dot{A}^2 + (2q-1)\frac{\dot{A}}{t} + (p+1)\dot{B}\left(\dot{A} + \frac{1}{t}\right)\right]\bar{g}_{ab}\,. \end{aligned} \tag{2.8}$$

Now we note that if we use the extremality condition similar to the static BPS p-brane,

$$(p+1)B + (q-1)A = 0 \tag{2.9}$$

then the components of the Ricci tensor given in (2.8) simplify to

$$\begin{aligned} R_{tt} &= -\left(\ddot{A} + \frac{q}{t}\dot{A} + \frac{(q-1)(d-2)}{p+1}\dot{A}^2\right) \\ R_{xx} &= e^{2B-2A}\left(\ddot{B} + \frac{q}{t}\dot{B}\right) \\ R_{ab} &= t^2\left(\ddot{A} + \frac{q}{t}\dot{A}\right)\bar{g}_{ab}\,. \end{aligned} \tag{2.10}$$

Here 'dot' represents the derivative with respect to t. Substituting R_{xx}, R_{ab}, R_{tt} in (2.2) and from the ϕ equation in (2.4) we get,

$$\ddot{B} + \frac{q}{t}\dot{B} + \frac{b^2(q-1)}{2(d-2)}\frac{e^{2(p+1)B+a\phi}}{t^{2q}} = 0 \tag{2.11}$$

$$\ddot{A} + \frac{q}{t}\dot{A} - \frac{b^2(p+1)}{2(d-2)}\frac{e^{2(p+1)B+a\phi}}{t^{2q}} = 0 \tag{2.12}$$

$$\ddot{A} + \frac{q}{t}\dot{A} + \frac{(q-1)(d-2)}{(p+1)}\dot{A}^2 + \frac{1}{2}\dot{\phi}^2 + \frac{b^2(q-1)}{2(d-2)}\frac{e^{2(p+1)B+a\phi}}{t^{2q}} = 0 \tag{2.13}$$

$$\ddot{\phi} + \frac{q}{t}\dot{\phi} + \frac{ab^2}{2}\frac{e^{2(p+1)B+a\phi}}{t^{2q}} = 0\,. \tag{2.14}$$

Comparing R_{xx} equation (2.11) and ϕ equation (2.14) we find,

$$\phi = \frac{a(d-2)}{q-1}B\,. \tag{2.15}$$

Now using the extremality relation (2.9), we note that R_{xx} and R_{ab} equations given in (2.11) and (2.12) are identical and take the form,

$$\ddot{B} + \frac{q}{t}\dot{B} + \frac{b^2(q-1)}{2(d-2)}\frac{e^{B\chi}}{t^{2q}} = 0 \tag{2.16}$$

where we have used eq. (2.15). Also, in the above $\chi = 2(p+1) + a^2(d-2)/(q-1)$. Now if $H(t)$ is a harmonic function[5] in the $(q+1) = (d-p-1)$-dimensional transverse space, then $H(t)$ satisfies,

$$\ddot{H} + \frac{q}{t}\dot{H} = 0\,. \tag{2.17}$$

Putting $e^B = H^\alpha$, where α is a constant to be determined from eq. (2.16), this equation reduces to a first order differential equation of H. Then H is determined from eq. (2.16) as,

$$H = \left[1 + \sqrt{\frac{b^2}{2(q-1)(d-2)\alpha}}\frac{1}{t^{q-1}}\right] \tag{2.18}$$

and the constant α is given as, $\alpha = -2/\chi$. Now since $\chi > 0$, we have $\alpha < 0$, therefore demanding the harmonic function H to be real we find $b^2 < 0$. In other words, the q-form field-strength $F_{[q]}$ given in (2.6) is purely imaginary. We therefore have,

$$e^{2B} = H^{2\alpha} = H^{-4/\chi}\,, \qquad e^{2A} = e^{-\frac{2(p+1)}{q-1}B} = H^{\frac{4(p+1)}{\chi(q-1)}}\,. \tag{2.19}$$

The solutions then take the form,

$$\begin{aligned} ds^2 &= H^{\frac{4(p+1)}{\chi(q-1)}}\left(-dt^2 + t^2 dH^2_{d-p-2}\right) + H^{-\frac{4}{\chi}}\left(dx_1^2 + \cdots + dx_{p+1}^2\right) \\ F_{[q]} &= b\mathrm{Vol}(H_q)\,, \qquad e^{2\phi} = H^{-\frac{4a(d-2)}{(q-1)\chi}} \end{aligned} \tag{2.20}$$

with 'b' being purely imaginary and H is as given in (2.18). It can be easily checked that this solution satisfies R_{tt} equation (2.13). The solutions (2.20) represent $(p+1)$-dimensional euclidean branes with imaginary magnetic charges and the imaginary charge is a result of the extremality condition (2.9). The solutions are given in the Einstein frame and have exactly the same form as the static BPS p-brane solutions of type-II string theories [18]. For various values of a, χ and q they represent different brane solutions of (dimensionally reduced) string/M theories. A list of these parameters for $d = 11$, 10 is given in table I of ref. [11]. The only case we have to be careful is when $q = 5$ in $d = 10$. Because in that case, the field-strength would be self-dual and is not considered in our solutions. For this case the dilaton coupling a would be zero and by self-duality $F^2_{[5]} = 0$. Therefore from the dilaton equation of motion (2.7), we see that it can be set to a constant. The field-strength in that case would take the form,

$$F_{[5]} = \frac{b}{\sqrt{2}}(1 + *)\,\mathrm{Vol}(H_5)\,. \tag{2.21}$$

The metric can be obtained from (2.20) by putting $a = 0$. We note that when $F_{[q]}$'s belong to RR sector of string theories, the solutions (2.20) would be real if we start from 'starred'

[5] We would like to point out that although we call this function as harmonic function (in analogy with static solutions) throughout this paper, it is not really a solution of $(q+1)$-dimensional Laplace equation in spherical coordinates. It is rather a solution of Laplace-like equation in hyperbolic coordinates, where, there are one time-like and q space-like coordinates. Also t above is not a radial coordinate. It is a time-like coordinate with $\mathrm{SO}(d-p-2,1)$ invariance as mentioned earlier.

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theory action. Then the solutions (2.20) would represent the Ep-branes of type-II* theories. However, when $F_{[q]}$'s belong to the NSNS sector, (2.20) would be real if the action contains kinetic terms of the NSNS sector gauge fields with signs opposite from those of the ordinary theories. But the relation between these theories and the ordinary type-II theories is not clear to us. Similar situation arises also for $d = 11$. In this case, $F_{[4]}^2$ term or $F_{[7]}^2$ term has opposite signs form that in ordinary M-theory. However, the dimensional reductions (along a space-like direction) of this theory leads neither to type-IIA theory nor to type-IIA* theory. So, as in the previous case the relation between this M-theory and ordinary M-theory or type-IIA/IIA* theory is not clear to us.

3. S-brane solutions in arbitrary dimensions

We have seen in the previous section that on imposing the extremality condition (2.9), the solutions of the equations of motion (2.2)–(2.4) become imaginary.[6] In this section, we will see that by relaxing the condition (2.9), we can get real solutions. We modify the condition (2.9) as follows,

$$(p+1)B + (q-1)A = \ln G(t)\,. \tag{3.1}$$

Now using (3.1) we obtain the non-zero components of the Ricci tensor from (2.8) as,

$$\begin{aligned}
R_{tt} &= -\ddot{A} - \frac{\ddot{G}}{G} + \frac{\dot{G}^2}{G^2} - \frac{1}{p+1}\left(\frac{\dot{G}}{G} - (q-1)\dot{A}\right)^2 - (q-1)\dot{A}^2 + \frac{\dot{G}}{G}\dot{A} - \frac{q}{t}\dot{A} && (3.2)\\
R_{xx} &= e^{2B-2A}\left(\ddot{B} + \frac{\dot{G}}{G}\dot{B} + \frac{q}{t}\dot{B}\right) && (3.3)\\
R_{ab} &= t^2\left(\ddot{A} + \frac{q}{t}\dot{A} + \frac{\dot{G}}{G}(\dot{A} + \frac{1}{t})\right)\bar{g}_{ab}\,. && (3.4)
\end{aligned}$$

Substituting these in (2.2) and (2.4), the R_{xx}, R_{ab}, R_{tt} and ϕ equations take the forms,

$$\begin{aligned}
\ddot{B} + \frac{q}{t}\dot{B} + \frac{\dot{G}}{G}\dot{B} + \frac{b^2(q-1)}{2(d-2)}\frac{e^{2(p+1)B+a\phi}}{G^2 t^{2q}} &= 0 && (3.5)\\
\ddot{A} + \frac{q}{t}\dot{A} + \frac{\dot{G}}{G}(\dot{A} + \frac{1}{t}) - \frac{b^2(p+1)}{2(d-2)}\frac{e^{2(p+1)B+a\phi}}{G^2 t^{2q}} &= 0 && (3.6)\\
-\ddot{A} - \frac{\ddot{G}}{G} + \frac{\dot{G}^2}{G^2} - \frac{1}{p+1}\left(\frac{\dot{G}}{G} - (q-1)\dot{A}\right)^2 - (q-1)\dot{A}^2 + \frac{\dot{G}}{G}\dot{A} - \frac{q}{t}\dot{A} - &\\
-\frac{1}{2}\dot{\phi}^2 - \frac{b^2(q-1)}{2(d-2)}\frac{e^{2(p+1)B+a\phi}}{G^2 t^{2q}} &= 0 && (3.7)\\
\ddot{\phi} + \frac{q}{t}\dot{\phi} + \frac{\dot{G}}{G}\dot{\phi} + \frac{ab^2}{2}\frac{e^{2(p+1)B+a\phi}}{G^2 t^{2q}} &= 0\,. && (3.8)
\end{aligned}$$

[6]If we restrict ourselves only to type-II theories this would mean that for the time dependent case there are no extremal solutions. However, here we are using the notion of extremality in the broader sense including the 'starred' theories. The function $G(t)$ defined below is called the non-extremality function also in this broader sense.

Using (3.1) into (3.6), we first convert this equation into an equation of the function $B(t)$ and then using (3.5) we find an equation involving the function $G(t)$ only (this gives a restriction on the form of the non-extremality function) as,

$$\ddot{G} + \frac{2q-1}{t}\dot{G} = 0\,. \tag{3.9}$$

The solution of this equation is,

$$G(t) = 1 \pm \frac{\omega^{2(q-1)}}{t^{2(q-1)}} \tag{3.10}$$

where ω is an integration constant. We have also assumed that as $t \to \infty$, $G(t) \to 1$. However, note that in this limit there are no real solutions (except the flat space) of the equations of motion as discussed in the previous section. Since here we are discussing the real solutions, there will not be any extremal limit for the solutions we obtain later. We also point out that the upper sign in (3.10) does not lead to a real solution and we consider only the lower sign. Now $G(t)$ can be split up as,

$$G(t) = 1 - \frac{\omega^{2(q-1)}}{t^{2(q-1)}} = \left(1 - \frac{\omega^{q-1}}{t^{q-1}}\right)\left(1 + \frac{\omega^{q-1}}{t^{q-1}}\right) = HH' \tag{3.11}$$

where $H(t)$ and $H'(t)$ are two harmonic functions in the $(q+1)$-dimensional transverse space satisfying equations of the form (2.17). We can now try to solve the equations for $B(t)$ in (3.5) in analogy with the previous section by choosing,

$$B = \alpha \ln H - \beta \ln H' \tag{3.12}$$

where α and β are two parameters to be determined from the equations of motion. Before using this form of B, we determine the form of ϕ in terms of B from equations (3.5) and (3.8). Using the last two equations we obtain,

$$\left(\ddot{\phi} - \frac{a(d-2)}{q-1}\ddot{B}\right) + \frac{q}{t}\left(\dot{\phi} - \frac{a(d-2)}{q-1}\dot{B}\right) + \frac{\dot{G}}{G}\left(\dot{\phi} - \frac{a(d-2)}{q-1}\dot{B}\right) = 0\,. \tag{3.13}$$

The solution to this equation is given by,

$$\phi = \frac{a(d-2)}{q-1}B + \delta \ln \frac{H}{H'} \tag{3.14}$$

where δ is an arbitrary constant and so,

$$e^{2(p+1)B + a\phi} = \left(\frac{H}{H'}\right)^{a\delta} e^{B\chi} \tag{3.15}$$

with $\chi = 2(p+1) + a^2(d-2)/(q-1)$ as given before. Now using (3.15) and (3.12) in (3.5) we see that this equation can be consistently solved if

$$\alpha = \frac{1 - a\delta}{\chi}, \qquad \beta = -\frac{1 + a\delta}{\chi}\,. \tag{3.16}$$

The solution then is,

$$\omega^{2(q-1)} = \frac{b^2\chi}{4(d-2)(q-1)}\,. \tag{3.17}$$

However, it can be easily checked that this solution does not satisfy the R_{tt} equation (3.7). The reason is we have not introduced enough number of parameters and the system of equations (3.5)–(3.7) become overdetermined. In order to get around this situation we will use a different ansatz for B (other than (3.12)) introducing more parameters such that R_{tt} equation (3.7) can be made consistent. So, we take the form of B as,

$$e^B = \left[\cos^2\theta\left(\frac{H}{H'}\right)^{\alpha} + \sin^2\theta\left(\frac{H'}{H}\right)^{\beta}\right]^{\gamma} = F^{\gamma} \tag{3.18}$$

where $F = [\cos^2\theta(H/H')^{\alpha} + \sin^2\theta(H'/H)^{\beta}]$. The justification for choosing this particular form of B can be understood as follows. We have seen before that $e^B = (HH')^{\alpha}(H')^{2a\delta/\chi}$ is inconsistent with the equations of motion. So, for $\delta = 0$, $e^B = (HH')^{\alpha}$ is inconsistent. With the two harmonic functions H and H', we can form two other functions i.e. H/H' and H'/H. So, e^B could be either $(H/H')^{\alpha}$ or $(H'/H)^{\beta}$, with α, β having the same sign or it could be a linear combination of both of them. The linear combination would introduce more parameters necessary for the consistency of the equations of motion. Also, if we insist that as $t \to \infty$, $e^B \to 1$ then the only possible linear combination is as given in (3.18). We have further raised this function to the arbitrary power γ. Note that if we do not insist $e^B \to 1$ as $t \to \infty$ then F could take the form $F = a_1^2(H/H')^{\alpha} + a_2^2(H'/H)^{\beta}$ and in that case instead of a single parameter θ, we introduce two parameters a_1 and a_2. Even in this case the equations of motion can be solved consistently. However, for simplicity we choose the form of e^B as in (3.18) and mention later how the solution would change with the additional parameter.

Now using the form of G (in eq. (3.11)), ϕ (in eq. (3.14)) and B (in eq. (3.18)) into eq. (3.5) we obtain,

$$\left[4\gamma(q-1)\omega^{2(q-1)}(\alpha+\beta)^2\sin^2\theta\cos^2\theta\right]\frac{H^{\alpha-\beta-2}H'^{\beta-\alpha-2}}{F^2} + \frac{b^2}{2(d-2)}H^{a\delta-2}H'^{-a\delta-2}F^{\gamma\chi} = 0\,. \tag{3.19}$$

We thus obtain from here

$$\begin{aligned} \gamma\chi &= -2\,, \qquad \alpha - \beta = a\delta \\ \omega^{2(q-1)} &= \frac{b^2\chi}{16(d-2)(q-1)(\alpha+\beta)^2\sin^2\theta\cos^2\theta}\,. \end{aligned} \tag{3.20}$$

We thus get a consistent real solution of the equations of motion (3.5), (3.6) and (3.8) in terms of four parameters α, δ, ω and θ. Next we need to check whether this solution is consistent with R_{tt} equation (3.7). Substituting this solution into (3.7) we get for consistency a relation between the parameters as,

$$\frac{1}{2}\delta^2 + \frac{2\alpha(\alpha - a\delta)(d-2)}{\chi(q-1)} = \frac{q}{q-1}\,. \tag{3.21}$$

Since using (3.21) we can eliminate one of the two parameters α or δ we therefore have real solutions with three independent parameters α, ω and θ, very similar to the solutions obtained by KMP in $d = 10$ [10]. So, the complete isotropic and localized S-brane solutions in d-dimensions can be written as,

$$\begin{aligned}
ds^2 &= F^{\frac{4(p+1)}{(q-1)\chi}} (HH')^{\frac{2}{q-1}} \left(-dt^2 + t^2 dH^2_{d-p-2}\right) + F^{-\frac{4}{\chi}} \left(dx_1^2 + \cdots + dx_{p+1}^2\right) \\
e^{2\phi} &= F^{-\frac{4a(d-2)}{(q-1)\chi}} \left(\frac{H}{H'}\right)^{2\delta} \\
F_{[q]} &= b\,\mathrm{Vol}(H_{d-p-2})
\end{aligned} \tag{3.22}$$

where F is given in eq. (3.18), H, H' are given in eq. (3.11) with ω given in (3.20). These solutions are characterized by four parameters α, δ, ω and θ with a relation between α and δ given in (3.21). If instead of taking e^B of the form (3.18), we had taken

$$e^B = \left[a_1^2 \left(\frac{H}{H'}\right)^{\alpha} + a_2^2 \left(\frac{H'}{H}\right)^{\beta}\right]^{\gamma} = F^{\gamma} \tag{3.23}$$

then the solutions will be characterized by an additional parameter and ω would be given as,

$$\omega^{2(q-1)} = \frac{b^2 \chi}{16(d-2)(q-1)(\alpha+\beta)^2 a_1^2 a_2^2}\,. \tag{3.24}$$

The solutions would then take exactly the same form as given in (3.22). Also, we would like to point out that in $d = 10$ and for $q = 5$, the field-strength is self-dual. This case is not included in our previous discussion and the equations of motion in this case need to be solved separately. However, the solutions can be obtained from (3.22) by setting $a = 0$ with the 5-form field-strength taking the form as given earlier in eq. (3.21) of section 2. Note that for $d = 11$, (3.22) would represent M-theory S-branes and in that case $\phi = 0$. So, from (3.14) we find that $a = 0$ and $\delta = 0$. When $\delta = 0$, $\alpha = \beta$ and eq. (3.21) will determine the value of the parameter α. M-theory S-branes will therefore have only two parameters ω and θ. But because of the mismatch of the number of parameters (string theory branes have three whereas M-theory branes have two parameters), the dimensional reductions of M-theory S-branes will not reproduce string theory S-branes contrary to our expectations. We have pointed out in ref. [20], that the isotropic and localized string theory S-branes can be obtained only from the delocalized (anisotropic) SM-branes by direct (double) dimensional reductions.

3.1 Relation with the KMP solution

In this subsection we will show that the solutions (3.22) match exactly with the KMP solutions [10] in $d = 10$. In $d = 10$, $a = (p-3)/2$, $q = 8-p$, $\chi = 32/(7-p)$ for the space-like Dp-branes and so, the solutions (3.22) take the forms,

$$\begin{aligned}
ds^2 &= F^{\frac{p+1}{8}} (HH')^{\frac{2}{7-p}} \left(-dt^2 + t^2 dH^2_{8-p}\right) + F^{-\frac{7-p}{8}} \left(dx_1^2 + \cdots + dx_{p+1}^2\right) \\
e^{2\phi} &= F^{\frac{3-p}{2}} \left(\frac{H}{H'}\right)^{2\delta} \\
F_{[q]} &= b\,\mathrm{Vol}(H_{8-p})
\end{aligned} \tag{3.25}$$

with $H = 1 - \omega^{7-p}/t^{7-p}$, $H' = 1 + \omega^{7-p}/t^{7-p}$ and F is as given in eq. (3.18). Also,

$$\omega^{2(7-p)} = \frac{b^2}{4(7-p)^2(\alpha+\beta)^2 \sin^2\theta \cos^2\theta}\,. \tag{3.26}$$

The parameters satisfy the relation,

$$\frac{1}{2}\delta^2 + \frac{\alpha\beta}{2} = \frac{8-p}{7-p} \tag{3.27}$$

where δ is given by $\alpha - \beta = a\delta$. Since the KMP metric is given in the string frame, we rewrite the metric in (3.25) also in string frame as,

$$ds^2 = F^{\frac{1}{2}} \left(\frac{H}{H'}\right)^{\frac{\delta}{2}} (HH')^{\frac{2}{7-p}} \left(-dt^2 + t^2 dH_{8-p}^2\right) + F^{-\frac{1}{2}} \left(\frac{H}{H'}\right)^{\frac{\delta}{2}} \left(dx_1^2 + \cdots + dx_{p+1}^2\right). \tag{3.28}$$

However, we note that this form of the metric is not quite the same as given in eq.(17) of ref. [10]. This is because the function F we have defined in (3.18) is not the same as $F_{\rm KMP}$ (see eq.(12) of ref. [10]) for the isotropic case. But we note that by defining

$$\begin{aligned} \alpha &= \frac{3n(p-3)}{2(7-p)} - \frac{m}{2} \\ \beta &= -\frac{3n(p-3)}{2(7-p)} - \frac{m}{2} \end{aligned} \tag{3.29}$$

where m, n are two parameters used in [10], we can write,

$$\begin{aligned} F &= \cos^2\theta \left(\frac{H}{H'}\right)^{\alpha} + \sin^2\theta \left(\frac{H'}{H}\right)^{\beta} \\ &= F_{\rm KMP} \left(\frac{H}{H'}\right)^{\frac{2(p-4)n}{7-p}}\,. \end{aligned} \tag{3.30}$$

Substituting (3.30) in (3.28) we find that the string-frame metric takes the form,

$$ds^2 = F_{\rm KMP}^{\frac{1}{2}} \left(\frac{H}{H'}\right)^{n\frac{p-1}{7-p}} (HH')^{\frac{2}{7-p}} \left(-dt^2 + t^2 dH_{8-p}^2\right) + F_{\rm KMP}^{-\frac{1}{2}} \left(\frac{H}{H'}\right)^{n} \left(dx_1^2 + \cdots + dx_{p+1}^2\right) \tag{3.31}$$

where we have used $\alpha - \beta = 6n(p-3)/(2(7-p)) = a\delta$, with $\delta = 6n/(7-p)$ and the dilaton takes the form,

$$e^{2\phi} = F^{\frac{3-p}{2}} \left(\frac{H}{H'}\right)^{2\delta} = F_{\rm KMP}^{\frac{3-p}{2}} \left(\frac{H}{H'}\right)^{pn}\,. \tag{3.32}$$

The parameter relation (3.27) now reduces to

$$9n^2(p+1) + m^2(7-p) = 8(8-p)\,. \tag{3.33}$$

This is precisely the same form of the isotropic and localized space-like Dp-brane solutions in $d = 10$ obtained in ref. [10].

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Now we show that the isotropic space-like NS-branes obtained in section 2.3 of ref. [10] also match with the solutions given in (3.22). We make an explicit comparison for the space-like NS5-brane (since this is magnetically charged) and then mention how we compare the space-like NS1-brane solution. For the NS5-brane we have $d = 10$, $p = 5$, $a = -1$, $q = 3$ and $\chi = 16$. The solution (3.22) therefore takes the form in string-frame,

$$\begin{aligned} ds^2 &= F(HH')\left(\frac{H}{H'}\right)^{\frac{\delta}{2}}\left(-dt^2 + t^2 dH_3^2\right) + \left(\frac{H}{H'}\right)^{\frac{\delta}{2}}\left(dx_1^2 + \cdots + dx_6^2\right) \\ e^{2\phi} &= F\left(\frac{H}{H'}\right)^{2\delta} \\ F_{[3]} &= b\,\mathrm{Vol}(H_3) \end{aligned} \tag{3.34}$$

with the parameter relation as given in (3.21). From (3.29) we now find $\delta = -3n$ and from (3.30), $F = F_{\mathrm{KMP}}(H/H')^n$. Substituting these in (3.34), we get the isotropic SNS5-brane solution as,

$$\begin{aligned} ds^2 &= F_{\mathrm{KMP}}(HH')\left(\frac{H}{H'}\right)^{-\frac{n}{2}}\left(-dt^2 + t^2 dH_3^2\right) + \left(\frac{H}{H'}\right)^{-\frac{3n}{2}}\left(dx_1^2 + \cdots + dx_6^2\right) \\ e^{2\phi} &= F_{\mathrm{KMP}}\left(\frac{H}{H'}\right)^{-5n} \\ F_{[3]} &= b\,\mathrm{Vol}(H_3)\,. \end{aligned} \tag{3.35}$$

This is precisely the isotropic SNS5-brane solution given in eq.(23) of ref. [10] with $k_2 = k_3 = k_4 = k_5 = k_6 = -n$, $k_1 + \tilde{k} = n$, $k_1 - \tilde{k} = m$. The space-like NS1-brane solution can be obtained from the SNS5-brane solution of (3.22) and applying the transformations $g_{\mu\nu} \to g_{\mu\nu}$, $\phi \to -\phi$, $F \to e^{\phi} * F$, $q \to d - q$ there (since this is electrically charged) and the solution then matches exactly with the isotropic SNS1-brane solution given in eq.(20) of [10]. Following a similar procedure M-theory S-branes obtained in [10] can also be shown to match with the solution (3.22).

3.2 Relation with the CGG solution

S-brane solutions in arbitrary dimensions have also been obtained by CGG in [11], but they used a different coordinate system from what we have used in this section. Although CGG solutions have the same symmetry $\mathrm{ISO}(p+1) \times \mathrm{SO}(d-p-2,1)$ of the metric, their solutions depend only on $\hat{t}$, which is the time coordinate of the transverse space and does not include the other $(d - p - 2)$ transverse space-like coordinates. On the other hand, the solutions we have described depend on the time-like coordinate $t = (\tilde{t}^2 - r^2)^{1/2}$ which includes all the $d-p-1$ transverse coordinates. However, since both the solutions have the same symmetry, it is reasonable to expect that there exists a coordinate transformation by which these solutions would map to each other. We will show that this is indeed true. But before we proceed, we should mention that the CGG solutions differ from the solutions we described in another respect. The CGG solutions are characterized by four parameters, whereas our solutions (3.22) are characterized by three parameters. The origin of this difference is that unlike in our case, where we assumed that metric becomes flat and $e^{2\phi} \to 1$ as $t \to \infty$, CGG solutions do not have this property. In order to compare our solutions with the CGG solutions, we will impose the same boundary condition in the CGG solutions and then the latter solutions will also have three parameters in them and the two solutions will become identical under a coordinate transformation.

The general S-brane solutions obtained by CGG are given as [11],

$$\begin{aligned} ds^2 &= \left[\sinh(q-1)\hat{t}\right]^{-\frac{2q}{q-1}} \left[\frac{(d-2)\chi\tilde{\alpha}^2}{(q-1)b^2}\right]^{-\frac{2(p+1)}{(q-1)\chi}} \left[\cosh\frac{\chi\tilde{\alpha}}{2}(\hat{t}-t_0)\right]^{\frac{4(p+1)}{(q-1)\chi}} e^{\frac{2a(p+1)}{(q-1)\chi}(c_1\hat{t}+c_2)} \times \\ &\quad \times \left(-d\hat{t}^2 + \sinh^2(q-1)\hat{t}dH_q^2\right) + \\ &\quad + \left[\frac{(d-2)\chi\tilde{\alpha}^2}{(q-1)b^2}\right]^{\frac{2}{\chi}} \left[\cosh\frac{\chi\tilde{\alpha}}{2}(\hat{t}-t_0)\right]^{-\frac{4}{\chi}} e^{-\frac{2a}{\chi}(c_1\hat{t}+c_2)} \left(dx_1^2 + \cdots + dx_{p+1}^2\right) \end{aligned} \tag{3.36}$$

$$e^{2\phi} = \left[\frac{(d-2)\chi\tilde{\alpha}^2}{(q-1)b^2}\right]^{\frac{2a(d-2)}{(q-1)\chi}} \left[\cosh\frac{\chi\tilde{\alpha}}{2}(\hat{t}-t_0)\right]^{-\frac{4a(d-2)}{(q-1)\chi}} e^{[2-\frac{2a^2(d-2)}{(q-1)\chi}](c_1\hat{t}+c_2)} \tag{3.37}$$

$$F_{[q]} = b\,\mathrm{Vol}(H_q)\,. \tag{3.38}$$

These solutions are characterized by five parameters $\tilde{\alpha}$, t_0, c_1, c_2, b with a relation between $\tilde{\alpha}$ and c_1 of the form,

$$\frac{(p+1)c_1^2}{\chi} + \frac{\chi\tilde{\alpha}^2(d-2)}{2(q-1)} = q(q-1)\,. \tag{3.39}$$

We will map this solution to (3.22) by a coordinate transformation given as,

$$\hat{t} = -\frac{1}{q-1}\ln\frac{H}{H'} \tag{3.40}$$

From (3.40) we obtain,

$$\left[\sinh(q-1)\hat{t}\right]^{-\frac{2q}{q-1}}\left(-d\hat{t}^2 + \sinh^2(q-1)\hat{t}\,dH_q^2\right) = \frac{(HH')^{\frac{2}{q-1}}}{(2^{\frac{1}{q-1}}\omega)^2}\left(-dt^2 + t^2\,dH_q^2\right). \tag{3.41}$$

Now comparing the transverse part of the metric in (3.36) and (3.22) and also using (3.41) we find that they match if we identify,

$$\begin{aligned} F &= \cos^2\theta\left(\frac{H}{H'}\right)^{\alpha} + \sin^2\theta\left(\frac{H'}{H}\right)^{\beta} = \cos^2\theta e^{-\alpha(q-1)\hat{t}} + \sin^2\theta e^{\beta(q-1)\hat{t}} \\ &\equiv \left[\frac{(d-2)\chi\tilde{\alpha}^2}{(q-1)b^2}\right]^{-\frac{1}{2}} \left[\cosh\frac{\chi\tilde{\alpha}}{2}(\hat{t}-t_0)\right] e^{\frac{a}{2}(c_1\hat{t}+c_2)} \left(2^{\frac{1}{q-1}}\omega\right)^{-\frac{(q-1)\chi}{2(p+1)}}\,. \end{aligned} \tag{3.42}$$

Note that with this identification the longitudinal parts of the metric in the two solutions also match if we rescale the coordinates x_i, for $i = 1, \ldots, p+1$ in (3.22) by $x_i \to x_i/(2^{1/(q-1)}\omega)^{(q-1)/(p+1)}$. We relate the parameters in the two solutions from (3.42) as,

$$\begin{aligned} \alpha &= \frac{1}{2(q-1)}(\chi\tilde{\alpha} - ac_1) \\ \beta &= \frac{1}{2(q-1)}(\chi\tilde{\alpha} + ac_1)\,. \end{aligned} \tag{3.43}$$

We also get from there,

$$\cos^2\theta = \frac{1}{2}e^{\frac{\chi\tilde{\alpha}}{2}t_0+\frac{ac_2}{2}}\left[\frac{(d-2)\chi\tilde{\alpha}^2}{(q-1)b^2}\right]^{-\frac{1}{2}}\left(2^{\frac{1}{q-1}}\omega\right)^{-\frac{(q-1)\chi}{2(p+1)}}$$
$$\sin^2\theta = \frac{1}{2}e^{-\frac{\chi\tilde{\alpha}}{2}t_0+\frac{ac_2}{2}}\left[\frac{(d-2)\chi\tilde{\alpha}^2}{(q-1)b^2}\right]^{-\frac{1}{2}}\left(2^{\frac{1}{q-1}}\omega\right)^{-\frac{(q-1)\chi}{2(p+1)}}. \qquad (3.44)$$

From (3.43) we obtain, be

$$\tilde{\alpha} = \frac{(q-1)}{\chi}(\alpha+\beta)$$

and

$$ac_1 = -(q-1)(\alpha-\beta)\,, \qquad \text{or} \qquad c_1 = -(q-1)\delta\,. \qquad (3.45)$$

Using (3.45) the parameter relation (3.39) reduces to

$$\frac{(p+1)c_1^2}{\chi} + \frac{\chi\tilde{\alpha}^2(d-2)}{2(q-1)} = q(q-1) \qquad \Rightarrow \qquad \frac{1}{2}\delta^2 + \frac{2\alpha(\alpha-a\delta)(d-2)}{\chi(q-1)} = \frac{q}{q-1}\,. \quad (3.46)$$

This is precisely the parameter relation we obtained in (3.21). On the other hand from (3.44) we obtain,

$$\tan\theta = e^{-\frac{\chi\tilde{\alpha}}{2}t_0}$$
$$2^{\frac{1}{q-1}}\omega = \frac{e^{\frac{a(p+1)}{(q-1)\chi}c_2}\left[\cosh\frac{\chi\tilde{\alpha}}{2}t_0\right]^{\frac{2(p+1)}{(q-1)\chi}}}{\left[\frac{(d-2)\chi\tilde{\alpha}^2}{(q-1)b^2}\right]^{\frac{p+1}{(q-1)\chi}}}\,. \qquad (3.47)$$

Comparing the dilaton expressions (3.37) and (3.22) we further find that they match provided,

$$\left(2^{\frac{1}{q-1}}\omega\right)^{\frac{a(d-2)}{p+1}} = e^{c_2}\,. \qquad (3.48)$$

Eliminating ω in the last relation in (3.47) and (3.48), c_2 gets fixed in terms of other parameters in the CGG solutions as,

$$c_2 = \frac{a(d-2)}{2(p+1)(q-1)}\ln\left[\frac{(q-1)b^2}{(d-2)\chi\tilde{\alpha}^2}\cosh^2\frac{\chi\tilde{\alpha}}{2}t_0\right]. \qquad (3.49)$$

This shows that if we map the CGG solutions to the solutions (3.22) then one of the parameters of the CGG solutions gets removed and we are left with three parameter solutions. The origin of this phenomenon is the fact that while mapping the CGG solutions to (3.22) we are imposing the same boundary condition (the metric becoming flat and $e^{2\phi}\to 1$ as $t\to\infty$) to the CGG solutions and this removes an additional freedom in the CGG solutions. Thus we have performed a complete mapping of the CGG solutions described by the parameters $\tilde{\alpha}$, c_1, t_0, b (with a relation between $\tilde{\alpha}$ and c_1) to the solutions obtained in (3.22) described by the parameters α, δ, ω, θ (with a relation between α and δ). The parameters in these two solutions are related by

$$\begin{aligned}
c_1 &= -(q-1)\delta \\
\tilde{\alpha} &= \frac{(q-1)(2\alpha - a\delta)}{\chi} \\
t_0 &= -\frac{2}{(q-1)(2\alpha - a\delta)} \ln \tan\theta \\
b &= 4\omega^{q-1}(2\alpha - a\delta)\sqrt{\frac{(d-2)(q-1)}{\chi}} \sin\theta\cos\theta\,.
\end{aligned} \tag{3.50}$$

Note that these relations are exactly the same in $d = 10$ as obtained in eq.(3.17) of ref. [12].

4. Another non-extremal euclidean brane solutions

In this section we will discuss another class of non-extremal euclidean brane (other that the one discussed in section 3) solutions of the equations of motion (2.2)–(2.4). As opposed to the solutions obtained in (2.2), these solutions will not be real and we will interpret the corresponding real solutions as the non-extremal E-brane solutions of type-II* theories when the form fields are in the RR sector. In the previous section we found from the consistency of the equations of motion that the non-extremality function $G(t)$ must be restricted by the equation (3.9). If we do not assume $G(t) \to 1$ as $t \to \infty$, then $G(t)$ could take the form,

$$G(t) = \frac{\omega^{2(q-1)}}{t^{2(q-1)}} = H^2(t) \tag{4.1}$$

where $H(t) = \omega^{q-1}/t^{q-1}$ is a harmonic function in the $(q+1)$-dimensional transverse space. One of the motivations for constructing such solutions was to look at the solutions (3.22) in the $t \to 0$ limit. It is known that for the static BPS D3-brane $r \to 0$ limit gives rise to the $AdS_5 \times S^5$ solution of type-IIB supergravity. Similarly for the time dependent solutions $t \to 0$ limit can be expected to give de Sitter type solution. However, it is clear from the solutions (3.22) that it is not possible to take $t \to 0$ limit directly as this will make $H = 1 - \omega^{q-1}/t^{q-1}$ negative and so the solutions will not remain real. One way to avoid the constant term in $G(t)$ (given in (3.10)) is to consider the solution of eq. (3.9) of the form given in (4.1). However, we will see that even this form of $G(t)$ does not lead to real solutions of type-II supergravities.

Now comparing B and ϕ equations (3.5) and (3.8) we obtain,

$$\phi = \frac{a(d-2)}{q-1} B\,. \tag{4.2}$$

Using the form of G in (4.1) and ϕ in (4.2) and assuming B to be of the form $B = \alpha \ln H$, where α is a parameter to be determined from the equations of motion, we find that eq. (3.5) reduces to,

$$\alpha(q-1)^2\omega^{2(q-1)} + \frac{b^2(q-1)}{2(d-2)} \frac{t^{2(q-1)}\omega^{\alpha\chi(q-1)}}{t^{\alpha\chi(q-1)}\omega^{2(q-1)}} = 0\,. \tag{4.3}$$

This equation can be solved if $\alpha\chi = 2$ and then the solution is

$$\omega^{2(q-1)} = -\frac{b^2\chi}{4(d-2)(q-1)} . \tag{4.4}$$

Note that since $\chi > 0$, the harmonic function H will be real only if $b^2 < 0$, or, b is purely imaginary. This implies that the field-strengths are purely imaginary. It can be easily checked that the R_{tt} equation (3.7) is automatically satisfied with this solution. Now since $\alpha = 2/\chi$ we have,

$$A = -\frac{p+1}{q-1}B + \ln H^2 = \ln H^{-\frac{2(p+1)}{(q-1)\chi}+2} \tag{4.5}$$

and so,

$$e^{2B} = H^{\frac{4}{\chi}} , \qquad e^{2A} = H^{-\frac{4(p+1)}{(q-1)\chi}+4} . \tag{4.6}$$

The complete solutions therefore are given as,

$$\begin{aligned} ds^2 &= H^{-\frac{4(p+1)}{\chi(q-1)}+4}\left(-dt^2 + t^2 dH^2_{d-p-2}\right) + H^{\frac{4}{\chi}}\left(dx_1^2 + \cdots + dx_{p+1}^2\right) \\ e^{2\phi} &= H^{\frac{4a(d-2)}{(q-1)\chi}} \\ F_{[q]} &= b\,\mathrm{Vol}(H_q) \end{aligned} \tag{4.7}$$

where b is purely imaginary. These solutions look very similar to the extremal solutions obtained in eq. (3.20). The powers of the harmonic functions in these two solutions differ by a sign and there is an additional H^4 factor in front of the transverse part of the metric in (4.7). This extra H^4 factor is due to the presence of the non-extremality function $G(t)$ which was absent for the solution (3.20). However, note that the harmonic functions $H(t)$ in these two solutions are different in general. They match only in the limit $t \to 0$. Also for the extremal solutions (3.20), $H(t) \to 1$ for $t \to \infty$ and the metric becomes flat, but for the solutions (4.7), $t \to \infty$ limit is ill-defined. Eq. (4.7) represent non-extremal euclidean branes and as discussed at the end of section 2, when the field-strengths belong to the RR sector, we can interpret the real solutions as E-branes of type II* theories.

5. Conclusion

To summarize, in this paper we have constructed various time dependent solutions of supergravity equations of motion containing a dilaton and a q-form field-strength in arbitrary dimensions. We have directly solved the non-linear equations of motion (unlike the method used in [10]) of the corresponding action. The metrics are assumed to have the symmetries $\mathrm{ISO}(p+1) \times \mathrm{SO}(d-p-2,1)$ and the field-strengths are assumed to be magnetic, so, they represent the $(p+1)$-dimensional magnetically charged euclidean branes. We found that when the metric components satisfy an extremality condition, similar to the static BPS p-branes, then the magnetic charges of these solutions become imaginary. But when the field-strengths belong to the RR sector, these solutions can become real if we think of them as the solutions of the so-called 'starred' theories instead of the ordinary type-II theories. In that case the solutions would represent Ep-branes of type-II* theories. We found that this problem does not arise if we relax the extremality condition and in that case we obtained real time-dependent solutions of the type-II supergravity equations of motion. These solutions are the generalizations of the supergravity Sp-brane solutions obtained by KMP (in $d = 10$) to arbitrary dimensions. We observed that in order to solve the equations of motion consistently we need to introduce at least three parameters, whose physical meanings are not clear to us. These solutions (as pointed out in [10]) have generic singularities at $t = \omega$, whose resolution is an important open problem to understand. We showed how our solutions exactly reduce to the solutions found by KMP in $d = 10$. Sp-brane solutions in a different coordinate systems were also obtained by CGG [11] in arbitrary dimensions. However, the solutions in [11] are characterized by four parameters instead of three as in our case. This difference is due to the use of different boundary conditions in these two sets of solutions. When we used the same boundary conditions we showed that the CGG solutions get mapped exactly to our solutions by a coordinate transformation given in (3.40). We have also given the relations between the parameters in these two solutions. Finally, we have obtained another class of non-extremal euclidean brane solutions. These solutions are not real and as before when the field-strengths belong to the RR sector, they can be made real by interpreting them as the non-BPS Ep-brane solutions of type-II* string theories. We pointed out similarities of these non-extremal solutions and the extremal solutions obtained in section 2.

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